Difference Principle and Black-hole Thermodynamics

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Abstract

The heuristic principle that constructive dynamics may arise wherever there exists a difference, or gradient, is discussed. Consideration of black-hole entropy appears to provide a clue for setting a lower bound on any extensive measure of such collective system difference, or potential to give rise to constructive dynamics. It is seen that the second-power dependence of black-hole entropy on mass is consistent with the difference principle, while consideration of Hawking radiation forces one to beware of implicit figure-ground distinctions in the application of the difference principle.

I. INTRODUCTION

In this paper a general physical principle, called the "difference principle", is presented. This principle may be used to gain insight into questions of dynamics in many contexts, for purposes of instruction or the conception of theory. The difference principle is an expression of the observation that difference in space (non-equilibrium) bears a reciprocal and self-propagating relation to difference in time (flow, or change).

The structure of the paper is as follows: Section II is a motivation and justification for presentation of "principles" in general (as contrasted to specific scientific theories). Section III comprises the main exposition via examples of the principle in its relation to the general field of non-equilibrium thermodynamics. Section IV is a discussion of the difference principle in relation to black-hole thermodynamics.

II. PHYSICAL LAWS, PRINCIPLES, AND LOGIC

Our interest in physics begins with innocent, reasonable questions like: How can a sailboat go upwind? How can an albatross keep flying, without ever beating its wings? How can a burning flame make the inside of a gas refrigerator cold? These are presumably the more advanced questions that follow the more fundamental questions of the type: What makes the wind blow? Why does heat go from hot places to cold places?

What's in an explanation? In his formal theory of inductive inference, Solomonoff proposed that scientific theory seeks information compression, and that scientific laws essentially summarize observations (including observations to come) in compact form.[1] With this view, the essence of science is to discover the correlations of the world; there is no mandate to provide account of a temporal causal sequence that comprises the history of an observation in need of explanation. Perhaps this is in consonance with the "Ithaca interpretation" of quantum mechanics that "correlations have physical reality; that which they correlate does not",[2] or Wheeler's "it from bit" program to discover the possible information-theoretic foundation of all physics.[3]

Classical physics, the physics of our experience and presumably the physics that influenced the evolution of our cognition, is evidently local, causal, and reversible at the microscopic level.[4] Perhaps for this reason we favor causal accounts of observations, even

though our everyday experience depends in a very essential way on statistical effects,[5] and even though the leading edge of theoretical physics forces us to consider that there may be no fundamental causal laws, but only probabilities for physical processes,[6] or that the ultimate causal laws may not be simple at all, so that the best we can do is to summarize the "emergent" patterns and rhythms that we observe, effectively assuming that the underlying microscopic dynamics, at the finest scale, are "random" for all practical purposes.[7]

Our preference for causal, local explanation is probably the reason for the predominance of Newton's vectorial mechanics (based on the vector quantities *momentum* and *force*) over the mathematically equivalent analytic mechanics of Leibniz, Euler, and Lagrange (based on the scalar quantities *energy* and *action* (see, for example, Lanczos[8]), to which Hamilton's action principle belongs.

When Feynman presented his alternative derivation of quantum mechanics based on path integrals, inspired by thinking about the action principle, he disavowed having presented any fundamentally new results, but noted that "there is a pleasure in recognizing old things from a new point of view".[9] There is also great heuristic value in this, insofar as familiarity with various points of view may lead to fundamentally new results.

Even if certain principles of physics are not considered to be laws or explanations in their own right, they may serve the purpose of guiding one to correct hypotheses. The principle that a beam of light should take the quickest (not the shortest) path between two given points, for example, does not provide a local, causal account of how the light is refracted or "knows" what path to take, but it leads to a correct formulation of geometric optics. The Le Chatelier-Braun principle, that a chemical balance responds to an externally-imposed change of temperature, pressure, etc., in such a way as to counteract the imposed change, offers no mechanistic account of the process, but qualitatively summarizes a class of generally observed behavior, leading to proper expectations regarding further observations, and to good guesses for the construction of specific hypotheses.

The principle of evolution by natural selection exemplifies the quasi-tautological nature of certain scientific principles. The biologist Lotka observed in 1922 that the principle of natural selection might be considered to be a physical principle.[10] When stated as "one should expect to observe that which is able to propagate into the future," the principle appears to be almost pure logic.

Notably, the second law of thermodynamics, which is closely related to the difference

principle that I discuss here, is regarded by many not to be a physical law at all, but rather to belong to the realm of logical scientific inference.[11,12,13,14] Penrose has addressed the question of time asymmetry between probabilistic inference applied to the past versus probabilistic inference applied to the future, and shown the second law of thermodynamics to be a consequence of an extraordinarily low-entropy state of the early universe. In this paper the assumption is made that there is a subtle but useful distinction to be made between epistemological content, on the one hand, and the ontological content of what Toffoli has called "substrate-universal behavior",[14] on the other. Whereas the "entropic dynamics" program of Caticha[15,16] aims to derive the laws of dynamics purely from rules of inference, here I use the phrase "entropy dynamics" to refer to the fundamental difference-reduction and difference-generation reciprocal processes of Nature that, after all, gave rise to our capacity for mental models and to our particular form of logic.

Analogy is the core of cognition, according to Hofstadter.[17] In learning and modeling our world, we construct larger and larger equivalence classes of phenomena in a process of conceptual unification that he calls "chunking". Following this view one could say that, just as there is pleasure in seeing old things in a new way, conversely there is pleasure in recognizing how new things belong to enlarged revisions of old equivalence classes.

Whether or not we can say exactly why, we soon grow accustomed to the equivalence class of phenomena comprising things that go "downhill", whether by freedom (diffusion) or by law (field). Later we might become familiar with the equivalence class of phenomena comprising things that go uphill whenever something else goes downhill. Such is the class of phenomena to which the opening questions refer. In constructing this equivalence class, we learn to appreciate that the discharge of one difference may be leveraged to generate another difference, and that in fact this reciprocal relation may be a prevailing theme in the natural world, as much as in the realm of human ingenuity.

III. THE DIFFERENCE PRINCIPLE IN ENTROPY DYNAMICS

> Where a difference of temperature exists, motive force can be produced.

This was the observation of Sadi Carnot,[18] one of the founders of thermodynamics, before any formal statement of the Second Law of thermodynamics, and before the introduction of the term "entropy" by Clausius. One might paraphrase Carnot's principle by saying that

the flow of heat "downhill" from a high-temperature source to a low-temperature sink can be leveraged to generate a mechanical potential.

Of course it is analogously true that where a difference of elevation exists (as of hydraulic head) motive force can be produced, or that where a difference of chemical potential exists, motive force can be produced.

In seeking to explain the *why* of dynamics, as well as of various sorts of organization or "order" apparent in our environment, it is customary to give an energy account. We might, for example, trace a certain chemical potential upon which our society depends back to the constructive activities of organisms dependent upon the free energy of the sun, which in turn derives from the free energy of gravitational potential. "The flow of energy through a system tends to organize that system"[19] is a well-known general principle.

One might observe the organizing effect of tidal flow in an estuary, and take account of its ultimate origin in angular momentum. But apparently the effect depends upon the *equilibration* of the *difference* in angular velocity between the Earth and the Moon; angular momentum per se is of less importance than the difference. Perhaps we would be justified to revise the above-mentioned energy-flow principle: "The equilibration of a difference through a system tends to organize that system". It is noteworthy that, from a structural point of view rather than from a functional point of view, *organization* is *differentiation* (including the creation of hierarchy).

A. Difference and Motive

> There can be no distinction without motive, and there can be no motive unless contents are seen to differ in value.

So began George Spencer-Brown's development of symbolic logic from more primitive concepts.[20] His work lay outside of physics as usually defined, yet the statement applies to physical processes, as a description of substrate-universal behavior.

We recognize that the latent motive inherent in the hydraulic head of a river derives from the energy of the sun, and we observe that the resulting flow of the river may generate a succeeding differentiation in the form of sorted alluvium in placer deposits. The discharge of the water's hydraulic (gravitational) potential might be used to generate electrical potential;

current from the electrical potential may drive a motor, which drives a pump, and so forth in a cascade of sequentially-orthogonal differences and flows.

Here "sequentially-orthogonal" is used to mean that the succeeding difference generated by the discharge of the preceding difference may appear in a state-space dimension orthogonal to that of the preceding difference, as for example electrical potential could be considered to be orthogonal to gravitational potential, because they are mutually independent, though each can be transformed into the other. Clearly the second law of thermodynamics precludes reversible transformation back and forth without net equilibration of difference, but the essential point is that the second law exhibits a constructive aspect in the operation of entropy dynamics—the complementary processes of equilibration of differences and generation of orthogonal differences.

"Asymmetry is the cause of phenomena," Pierre Curie said.[21] Asymmetry is essentially difference. Symmetry, Rosen shows us, is akin to entropy.[22] Some reflection suggests that Carnot's principle could have been stated with greater generality, as an expression of the difference principle of entropy dynamics:

> Where a difference exists, motive force can be produced.

The principle is not, after all, essentially dependent upon the specific nature of the primary difference.

It also happens that a difference of temperature can, by way of heat flow, produce an electrical potential (the so-called Seebeck effect), while a difference of electrical potential can, by way of electrical current, produce a thermal potential (the so-called Peltier effect); Kelvin conjectured, and Onsager confirmed, that these reciprocal relations were quantitatively symmetrical.[23]

A more complete expression of the difference principle of entropy dynamics might therefore be given as:

> Where a difference exists, motive force can be produced, and where a motive force exists, a difference can be produced.

Spatial and temporal difference appear as self-propagating duals. The use of the term "dual" is intended to echo Toffoli's use of the word when he conjectured that *entropy* is a concept dual to that of *action*:[24] as *entropy* is interpreted as "uncertainty as to state", so *action* is

interpreted to be "uncertainty as to trajectory". Likewise I conjecture that any measure of collective internal *difference* of state, that is, any measure of non-equilibrium or asymmetry, should be associated with a dual measure of *change*, or that collective characteristic of trajectories that would imply the subsequent generation of further non-equilibrium.

B. Application of the Difference Principle

So how can a sailboat go upwind? The value of the difference principle in framing the problem is that it draws one's attention to the asymmetry inherent in a system comprising air that is moving and a substrate that is not. It leads to the expectation that this asymmetry could be leveraged to generate orthogonal asymmetry, and in a sequence of at least two steps, to produce a "flow" opposite the original gradient.

While the standard vector analysis of resultant force on a sail together with the reaction of the keel explains the mechanics of the mystery of making way against the wind, it helps little toward understanding that one could just as well build a little windmill-driven car that could advance over the ground directly upwind. By reference to the difference principle, this possibility would be obvious. Of course the "efficiency" of such transformations of asymmetry are necessarily less than 100%, so the little car could not be expected to advance by its own induced headwind!

How can an albatross keep flying, without ever beating its wings? Understanding the difference principle, one might realize that, whereas there is no inherent "difference" in a constant wind field, if there is a wind gradient, then "motive force" can be produced.

Suppose that there is a horizontal gradient, with the wind velocity increasing with altitude above the seas. Then the albatross could descend in the downwind direction, and ascend in the upwind direction (by its own inertia), encountering both ways an increasing supply of wind, so to speak, to make up for its loss of airspeed due to friction. Hence a surplus of lift might be gained, and the bird's sum of potential and kinetic energy maintained or increased, at the expense of the wind gradient (which gradient should be expected to decrease slightly, reducing the wind speed aloft compared to that below).

Alternatively one might imagine riding in a hot-air balloon in such a horizontal wind gradient, with a little wind generator on board. Your only control of the balloon is to ascend or descend. If you maintain constant altitude, you feel no relative wind, and your

wind generator can extract no work from the environment. It makes no difference to you for this purpose that you may be moving over the ground. But whenever you ascend or descend, your wind generator works: you encounter the difference in the wind field, and a motive force can be produced.

How can a burning flame make the inside of a gas refrigerator cold? Again, the way that the difference principle might help to frame the problem of understanding absorption-cycle refrigeration is to recognize that the process must, as always, involve an original source-and-sink difference, and it must involve at least two orthogonal differences, or asymmetries. In the light of familiarity with the difference principle in relation to entropy dynamics, there is no reason to doubt that a temperature difference could produce another temperature difference, via intermediate transformations of asymmetry. In this case the orthogonal intermediate asymmetry is that of chemical species separated by heating. When this difference is discharged, by allowing the species to mix, this flow can pump heat, producing the difference in temperature between the inside of the refrigerator and the outside.

So-called "transitional dynamics" are exploited by fish in their swimming, by bees in their wing aerodynamics, and by engineers designing bio-inspired propellors (see, for example,[25,26,27]): fish seek vortices and other irregularities in water flow that they may leverage to their advantage, while bees and underwater-vehicle propulsion engineers achieve greater aerodynamic- or hydrodynamic efficiency with foils operating across the dynamic flow-regime boundary of stall. Even stochastic resonance may be seen to fit the "difference principle" paradigm; as Berdichevsky and Gitterman observe, "Common to ratchets and stochastic resonance is their ability to extract useful work from a fluctuating environment".[2838]

As a further illustration of the "difference principle" way of looking at a problem, I offer the following example of a process involving sequentially-orthogonal differences propagated via the duals *potential* and *flow*. When an electrical engineer has available a certain dc electrical potential and would like to produce a higher dc electrical potential, one scheme that would come to mind would be to arrange a circuit wherein a changing current, which is of course discharge of electrical potential, produces a changing magnetic potential (orthogonal to the electrical potential). The changing magnetic potential in turn produces the secondary electrical potential. In simple practical terms, the circuit is an oscillator and a transformer followed by a rectifier (an electrical ratchet). The value of the difference principle is simply

that it guides one to expect that the solution should exist, and that it may involve a two-step sequence of orthogonal differences.

In contrast, a similar example that does not appear to involve orthogonal differences is the case of the water ram. Here relatively low hydraulic potential is discharged, through a pipe, and the flow is periodically arrested, so that the inertial "water hammer" effect can be used, with a check valve (water ratchet), to produce relatively high hydraulic potential. But the secondary hydraulic potential can be considered to be orthogonal to the primary hydraulic potential insofar as the two potentials are independent (aside from their coupling through the mechanism itself).

IV. BLACK-HOLE THERMODYNAMICS

In the Penrose account of entropy dynamics, the original great state of non-equilibrium of the early universe was its uniform distribution of mass-energy (near-zero Weyl tensor of space-time curvature), providing, in effect, maximal collective gravitational potential.[29] In the interest of finding a theoretical lower bound for collective internal system "difference", non-equilibrium, or potential for further constructive dynamics, we try to imagine the opposite of uniform distribution of mass-energy, which would be singular concentration of mass-energy: we consider black holes, following the lines of thought of Bekenstein[30] and Lloyd[31]. The presumption is that a black hole can be taken as the physical realization of a "dead battery"[39], compared to the "fully-charged battery" of the low-entropy early universe configuration[40].

A. Second-power mass relation

The Bekenstein-Hawking entropy of a black hole S_{BH} is given by

$$S_{BH} = \frac{kAc^3}{4G\hbar}, \tag{1}$$

where k is Boltzmann's constant, A is the area of the event horizon, c is the speed of light, G is the gravitation constant, and $\hbar$ is Planck's constant divided by 2π. As shown by Bekenstein,[32] the entropy of a black-hole is proportional to its surface area.

Since the Schwarzschild radius R_S is given by

$$R_S = \frac{2GM_{BH}}{c^2}, \tag{2}$$

where M_{BH} is the mass, combining eq. (1) and eq. (2), and taking $A = 4\pi R_S^2$, the black-hole thermodynamic entropy can be expressed as a function of mass:

$$S_{BH} = \frac{4\pi k G M_{BH}^2}{\hbar c}. \tag{3}$$

Thus the thermodynamic entropy is proportional to the *square* of the mass (or energy).

Boltzmann's equation relates thermodynamic entropy S to information by

$$S = k\ln(W), \tag{4}$$

where W is the number of microstates consistent with the macrostate for which entropy is to be quantified; $\ln(W)$ is therefore the information (uncertainty as to state) in "nats", and dividing this by $\ln(2)$ would express the information in bits. Hence combining eq. (1) and eq. (4), the information entropy of a black hole I_{BH} in bits, as a function of the event horizon area A, is given (see for example Wheeler[3]) by

$$I_{BH} = \frac{Ac^3}{4\ln(2)G\hbar}, \tag{5}$$

or as a function of the mass M_{BH} by

$$I_{BH} = \frac{4\pi G M_{BH}^2}{\ln(2)\hbar c}. \tag{6}$$

It seems a bit surprising that the upper bound of the (missing) information content of a system would be proportional to the area of its boundary, and to the *square* of its mass, or energy, which might otherwise be considered to be the natural choice for a measure of system "size". After all, the upper bound for the speed of dynamical evolution of a system has been shown by Margolus and Levitin to be directly proportional to the average energy of the system:[33]

$$f \leq \frac{E}{\pi\hbar}, \tag{7}$$

where f is the number of mutually orthogonal changes per second, and E is the average system energy.

Perhaps the most surprising consequence of the second-power relation between mass and black-hole entropy, as noted by Bekenstein, is that the entropy of a black hole formed by

merging two separate black holes is greater than the sum of the separate black hole entropies, for the simple reason that $(M_1 + M_2)^2 \geq M_1^2 + M_2^2$: the entropy of the single black hole resulting from the merging of two similar, separate, black holes is double the sum of the entropies of the two black holes taken separately.

A physical consequence of this is that work could be done by merging two black holes, perhaps by extracting energy from gravitational waves.[34] This seems as odd as if one could wring further life out of two dead batteries by connecting something between them. And it seems to fly in the face of the "difference principle", or its converse, which would state, after the Spencer-Brown statement "there can be no motive unless contents are seen to differ in value", that

> Where no difference exists, no motive force can be produced.

But in fact careful consideration of the difference principle reveals that it would lead to just the correct expectation, because it refers to any *internal* difference inherent to the system under consideration. Whereas a single black hole, considered as a system (bounded perhaps by its event horizon), apparently has exhausted its potential for further constructive evolution, a pair of black holes must be considered as a larger system including the very difference posited by the existence of two distinguishable black holes.

The difference principle would lead one to expect that, in a system comprising more than one object, or indeed in a space comprising two distinguishable regions, there would be the possibility for further change—a "motive force" could be produced. If one tentatively hypothesized that black-hole entropy, as a sort of negative proxy for system internal non-equilibrium, were proportional to the first power of mass, and if one accepted the conservation of mass, then one would find in the merging of two black holes that the difference principle would be violated, insofar as the prior existence of two distinguishable black holes "made no difference" when they merged to become one; entropy remained constant. Guided then by the difference principle to expect a non-linear functional relation f between mass M_{BH} and entropy S_{BH} for which

$$f(M_1 + M_2) \geq f(M_1) + f(M_2), \tag{8}$$

one might be led to the simple guess that

$$S_{BH} \propto M_{BH}^2. \tag{9}$$

B. Hawking radiation

A more serious challenge to the applicability of the difference principle to black-hole thermodynamics is Hawking radiation. Thermodynamic entropy has the units of energy divided by temperature; if an object has energy and entropy, one might expect that the object should also have a temperature. Hawking was thus led to the discovery that a black hole does have a characteristic temperature, and can be expected to emit black-body radiation corresponding to this temperature.[35]

The temperature T_{BH} associated with a black hole is given by Hawking:[36]

$$T_{BH} = \frac{\kappa\hbar}{4\pi k}, \tag{10}$$

where κ is the surface gravity, which we will take to be given by

$$\kappa = \frac{GM_{BH}}{R_S^2}, \tag{11}$$

so that, combining eqs. (2), (10), and (11), black hole temperature may be expressed as a function of mass:

$$T_{BH} = \frac{c^4\hbar}{16\pi kGM_{BH}}. \tag{12}$$

Therefore T_{BH} is inversely proportional to M_{BH}, and the black hole is seen to have negative specific heat.[35]

The black hole was presumed to be at maximum entropy for a system of that mass—"exhausted" so to speak, with respect to the possibility for further constructive evolution—yet evidently there was the potential for an encore: it could explode![36] In the absence of accretion from its surroundings, a black hole emits Hawking radiation at an increasing rate, since it has negative specific heat and its temperature increases as it sheds mass. Even in the absence of accretion from its surroundings, it turns out that a black hole of one solar mass could endure longer than the present age of the universe (its theoretical lifetime being about 10^{64} years,[36]) but a black hole of 1kg mass would blow up in a matter of 10^{-21} second.[37] Certainly such a 100-megaton explosion represents further dynamics, possibly even constructive dynamics in the sense that this flux of energy, this "motive force", is likely to produce a new difference somewhere in the whole system considered.

"The whole system considered" is the key phrase to the resolution of the apparent paradox of Hawking radiation and black-hole explosions in the face of the difference principle. For in

considering Hawking radiation, the whole system considered necessarily includes the black hole *together with* its surroundings, i.e. space-time outside its event horizon.

It was supposed that the black hole of given mass might represent a lower bound for a measure of the constructive evolvability, due to internal differences, of a system of that mass. That may be the case, if there is any way to consider a black hole as an isolated system. But the picture of a 1kg black hole, with temperature of the order $10^{31}K$ and rising, as a *figure* against a *background* of relatively cold, empty space, is a picture of extreme difference. Again, the difference principle actually leads one correctly to expect that a great deal of constructive dynamics are likely to ensue.

V. CONCLUSION

The difference principle of entropy dynamics has been presented as a conceptual framework to provide insight into problems of dynamics in general. As "difference" is a fundamentally information-theoretic concept, this constitutes a viewpoint founded in information accounting rather than in energy accounting. Just as *entropy*, as multiplicity of possible states, and *action*, as multiplicity of possible paths, can be regarded as duals, so *difference*, as a collective measure of system non-equilibrium, and *flow*, as a collective measure of system change, may be regarded as self-propagating duals. It has been shown that the difference principle may provide some intuitive understanding and possible insight even when pushed to its limit in the strange realm of black-hole thermodynamics.

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[38] Stochastic resonance is the effect that a signal lying below the threshold of detection may become detectable by the addition of noise.

[39] What Americans call a “dead” battery, others call a “flat” battery, quite appropriately, since “flat” describes a high-entropy distribution.

[40] Interestingly, *flat* space has the least entropy, presumably for combinatorial reasons.